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Let $1/a^2$, $1/b^2$, $1/c^2$, be the numbers. Then $a^2+c^2=2b^2$, and solutions are given by

$$a=m^2-n^2+2mn$$
$$c=n^2-m^2+2mn$$
$$b=m^2+n^2.$$

To make these solutions complete, however, it is necessary to show that the formulae for a, c, and b give all the possible solutions. This may be proved as follows:

If $a^2+c^2=2b^2$ then b must be of the form m^2+n^2, for every divisor of the sum of two squares is itself the sum of two squares. Now

$$2b^2=2(m^2+n^2)^2=(1+1)(m^2+n^2)^2$$

can be expressed as the sum of two squares in but one way, the following:

$$(m^2-n^2-2mn)^2+(m^2-n^2+2mn).$$

Hence $a=m^2-n^2+2mn$, and $c=n^2-m^2+2mn$ give all the solutions of the problem.

Also solved by *L. C. WALKER.*

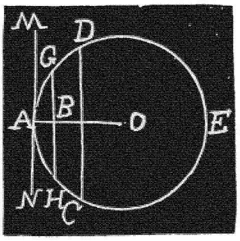
AVERAGE AND PROBABILITY.

109. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

A cylinder pierces a sphere in such a manner that the cylinder is tangent internally to the projection of the sphere in the plane xy. Find (1) the average surface, (2) the average volume of the sphere included within the cylinder.

Solution by LON C. WALKER, A.M., Professor of Mathematics, Petaluma High School. Petaluma, Cal.

Let $ACED$ be a section of the sphere through the center and the axis of the cylinder, O the center of the sphere, GH the axis of the cylinder, MN an element of the cylinder tangent to the sphere at A. Let S, V, be the required average surface and volume, respectively.



Put $OA=a$, $AB=r$. Now if we take AO for the axis of x, AM for the axis of z, and the perpendicular to OAM at A for the axis of y, the equations of the sphere and cylinder are respectively,

$$x^2+y^2+z^2=2ax, \text{ and } x^2+y^2=2rx.$$

$$\therefore\ S=\frac{2\int_0^a\int_0^{2r}\int_0^{\surd(2rx-x^2)}\dfrac{adrdxdy}{\surd(2ax-x^2-y^2)}}{\int_0^a dr}=\pi\int_0^a\int_0^{2r}drdx=\pi a^2,$$

the area of a great circle of the sphere.

If $y'=\surd(2ax-x^2)$, $z=\surd(2ax-x^2-y^2)$, then we have

$$V=\frac{4}{a}\int_0^a\int_0^{2r}\int_0^{y'}\int_0^{z'}drdxdydz=\frac{4}{a}\int_0^a\int_0^{2r}\int_0^{y'}(2ax-x^2-y^2)drdxdy$$

$$=\frac{2}{a}\int_0^a\int_0^{2r}\Big[(2ax-x^2)\sin^{-1}\Big(\frac{2r-x}{2a-x}\Big)^{\frac{1}{2}}+x\surd[2(a-r)(2r-x)]drdx$$

$$=\frac{8}{9a}\int_0^a\Big[3a^3\tan^{-1}\Big(\frac{r}{a-r}\Big)^{\frac{1}{2}}-(3a^2+2ar-8r^2)\surd(ar-r^2)\Big]dr=\tfrac{5}{6}a^3\pi$$

$=\frac{5}{8}$ of the volume of the sphere.

Also solved by *G. B. M. ZERR*, who gets $\frac{1}{2}\pi R^3$ as a result for the second part of the problem.

A partial solution was received from F. P. Matz.

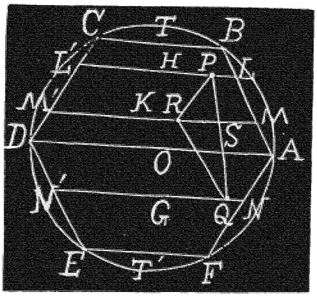
Professor Walker should have received credit in the last issue for solution of problem 108.

110. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Find the average area of the triangle formed by joining three random points taken on the surface of a regular hexagon, two on one side of a diagonal and the third on the other side.

Solution by the PROPOSER.

Let $ABCDEF$ be the hexagon; P, R the random points above the diagonal AD; Q the random point below the diagonal. Through P, R, Q draw LL', MM', NN' parallel to AD, and TT' perpendicular to AD through O. It is only necessary to consider the relative positions in which the line MM' lies between LL' and NN'.



Let $CB=OA=a$ be the side of the hexagon, $OH=u$, $OG=v$, $OK=w$, $HP=x$, $GQ=y$, $KR=z$, $KS=t$, $HL=x'$, $GN=y'$, $KM=z'$.

Then $OT=\frac{1}{2}a\surd3=u'$, $x=(a\surd3-u)/\surd3$, $y'=(a\surd3-v)/\surd3$, $z'=(a\surd3-w)/\surd3$, $t=y-[(y-x)(v+w)]/(u+v)$.

Area $PQR=\frac{1}{2}(t-z)(u+v)=A$, when $t>z$.

Area $PQR=\frac{1}{2}(z-t)(u+v)=A_1$, when $t<z$.

The limits of u are 0 and $\frac{1}{2}a\surd3$; of v, 0 and $\frac{1}{2}a\surd3$; of w, 0 and u; of x, $-x'$ and x'; of y, $-y'$ and y'; of z, $-z'$ and t, and t and z'.